

# HARDY–LITTLEWOOD FRACTIONAL MAXIMAL OPERATORS ON HOMOGENEOUS TREES

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ABSTRACT. Let  $T$  be a homogeneous tree equipped with the standard graph distance and the counting measure, and denote by  $\mathcal{M}^\gamma$ ,  $\gamma \in (0, 1]$ , the Hardy–Littlewood fractional maximal operator acting on complex valued functions defined on  $T$ . It was proved by Cowling, Meda, and Setti and, independently, by Naor and Tao, that the classical maximal operator  $\mathcal{M} = \mathcal{M}^1$  is of weak type  $(1, 1)$  and bounded on  $L^p(T)$  for any  $p > 1$ . This is a remarkable result since  $T$  is not a space of homogeneous type and the classical theory does not apply. In this talk we discuss the mapping properties of  $\mathcal{M}^\gamma$  between Lorentz and Lebesgue spaces on  $T$ , for values of  $\gamma \in (0, 1)$ . We will provide both positive and negative results, and we will highlight which problems remain open and comment on them. The talk is based on a joint work with Federico Santagati.

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