FOSCHIANS RARELY MAXIMIZE FOURIER EXTENSION ESTIMATES TO CONES

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Abstract. We consider the general Fourier extension estimate to the d-dim. cone in $\mathbb{R}^{1+d}$:

$$\|\widehat{f\mu}\|_{L^q(\mathbb{R}^{1+d})} \le C_{d,p}\|f\|_{L^p(d\mu)}.$$

Firstly, we prove that it admits *maximizers*, i.e. functions f that attain the best possible constant $C_{d,p}$. For $p = 2$, this estimate coincides with the *Strichartz estimate* for the wave equation; in this case, Foschi found that $f(\tau, \xi) = \exp(-\tau)$ is a maximizer for $d \in \{2, 3\}$. We further prove that such *foschian functions* have a chance of being maximizers for general d if and only if $p = 2$. This parallels the analogous result of Christ and Quilodrán that gaussian functions rarely maximize the Fourier extension estimate to the paraboloid. Our proof is entirely different, and it is based on the conformal compactification of $\mathbb{R}^{1+d}$ given by the Penrose transform.

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